Algebra 2, Quarter 1, Unit 1.1

Understanding Basic Parts of Expressions and Units

Overview

Number of instructional days: 5 (1 day = 45–60 minutes)

Content to be learned

- Use appropriate units in word problems.
- Use appropriate units in formulas.
- Select appropriate scale in graphs.
- Select appropriate scale in data displays.
- Understand the meaning of the origin in graphs.
- Understand the meaning of the origin in data displays.
- Interpret expressions in context in real-world, real-life problems.
- Understand the parts of expressions.
- Define quantities for modeling.
- Rewrite expressions into equivalent forms.
- Use different levels of accuracy when appropriate to answer word problems and real-life problems.

Mathematical practices to be integrated

Model with mathematics.

- Create graphs and data displays with appropriate units and scales to represent problems.

Use appropriate tools strategically.

- Change scale in graphing calculator views from various word problems involving the creation of graphs.
- Change scales and units in data displays.

Attend to precision.

- Decide the level of accuracy in data or world problems to represent answers appropriately.

Essential questions

- What are the similarities and differences in selecting scales in graphs and data displays?
- How do you know when units are appropriate for real-world situations?
- Why are units important in the problem-solving process?
- How do you determine the appropriate level of accuracy for a quantitative solution?
- What type of real numbers would a student find in applying the arithmetic operations of addition and multiplication to any combination of rational and irrational numbers?

Written Curriculum

Common Core State Standards for Mathematical Content

Quantities★	**N-Q**

Reason quantitatively and use units to solve problems.

N-Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.★

N-Q.2 Define appropriate quantities for the purpose of descriptive modeling.★

N-Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★

Seeing Structure in Expressions	**A-SSE**

Interpret the structure of expressions

A-SSE.1 Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*

A-SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grade 1, students measured objects that have the same units. In grade 2, students measured and estimated lengths in standard units. In grade 3, students solved problems involving measurement and estimation. In grade 4, students solved problems involving measurement and conversion of measurement from larger units to smaller ones.

In grade 5, students represented and interpreted data and converted measurements, they also graphed points on the coordinate plane. In grade 6, students represented and analyzed relationship between variables. In grade 7, students generated equivalent expressions. In grade 8, students constructed scatterplots. In algebra I, students reasoned quantitatively and solved problems, interpreted the structure of expressions, modeled relationships between two quantities, interpreted expressions in terms of modeled situations, and represented data on single count variable and two categorical data. Rewriting expressions into equivalent forms is a fluency in algebra 1. In geometry, students used volume formulas to solve problems.

Current Learning

Students use appropriate units in word problems and in formulas. Students select appropriate scale in graphs and in data displays. Students work with the meaning of the origin in graphs and in data displays. Students interpret expressions in context with real-world, real-world problems. Students understand the parts of expressions and how to use quantities for modeling. Students answer real life and world problems with various accuracies when appropriate.

Future Learning

In precalculus, students will use appropriate units in trigonometric answers of radian versus degree measure. In addition, calculus students will use appropriate units in washer and or disk method in addition to area under the curve calculations. Using appropriate units and scales in data representation will be used in various careers, such as temperature readings in food service careers, length, weight, area, and volume in construction, yet other examples will be used in utilities such as electrical current, flow rate, volumes of wastes or water, and runoff.

Additional Findings

Students struggle with understanding that a line of best fit is not appropriate for all data. In this unit, students will deal with real data that may have a weak or no correlation. "When doing experiments or dealing with real data, students may encounter "messy data," for which a line or a curve may not be an exact fit. They will need experience with such situations and assistance from the teacher to develop their ability to find a function that fits the data." (*Principles and Standards for School Mathematics*, p. 28)

Algebra 2, Quarter 1, Unit 1.2

Using Complex Number Systems

Overview

Number of instructional days: 5 (1 day = 45–60 minutes)

Content to be learned

- Know the complex number form $a + bi$.
- Know that $i = –1$.
- Use the commutative, associative, and distributive properties in adding, subtracting, and multiplying complex numbers.
- Use the quadratic formula to solve quadratic equations with complex solutions.
- Find zeros or roots for quadratic equations with complex roots.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Understand complex numbers.
- Know $i^2 = –1$ and $i\sqrt{1}$.

Look for and make use of structure.

- Add, subtract, and multiply complex numbers.
- Use properties of complex numbers.

Look for and express regularity in repeated reasoning.

- Use derivative to demonstrate complex solutions.
- Use the quadratic formula to solve quadratic equations for complex solutions.

Essential questions

- What do complex numbers represent?
- When are complex numbers important in your mathematical studies?
- What are the similarities and differences of using the quadratic formula with real numbers and complex numbers?
- How do the patterns of i help simplify complex numbers?
- What are the similarities and differences between the real number system and the complex number system?

Written Curriculum

Common Core State Standards for Mathematical Content

The Complex Number System	**N-CN**

Perform arithmetic operations with complex numbers.

N-CN.1 Know there is a complex number i such that $i^2 = –1$, and every complex number has the form $a + bi$ with a and b real.

N-CN.2 Use the relation $i^2 = –1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Use complex numbers in polynomial identities and equations. [*Polynomials with real coefficients*]

N-CN.7 Solve quadratic equations with real coefficients that have complex solutions.

*N-CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

**Teacher note: Algebra 2 PAP objective (Optional for Algebra 2 Regular)*

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

When in kindergarten, students learned addition and subtraction using manipulatives. After students went to first grade, they learned place value understanding and properties of operations. They solved problems using addition and subtraction. They began to understand and apply the properties of operations and the relationship between addition and subtraction.

In grade 3, students represented and solved problems involving addition and subtraction. Third graders used equal groups of objects to gain foundations for multiplication. They developed an understanding of fractions as numbers. Third graders also used all four operations to solve problems. Fourth graders extended their understanding of fraction equivalence and built fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

In grade 5, students used equivalent fractions as a strategy to add and subtract fractions. They wrote and interpreted numerical expressions. In grade 6, students applied and extended their previous understanding of multiplication of fractions by fractions. Seventh grade used properties of operations and generated equal expressions. Eighth graders worked with radical and integer exponents. The defined, evaluated, and compared functions. They also worked with square and cube roots.

In algebra I, students learned to factor quadratic expressions to reveal zeros, and they solved quadratic equations with real roots. They also used the distributive, associative, and commutative properties to add, subtract, and multiply polynomials.

Current Learning

Finding the zeros of complex numbers and the Fundamental Theorem of Algebra are critical areas of algebra 2. During this unit, students will learn $i=\sqrt{-1}$ and $i^2=-1$. They will recognize complex numbers in the form of $a + bi$. Students will use the commutative, associative, and distributive properties in addition, subtraction, and multiplication of complex numbers. After learning complex numbers, the students use the quadratic formula to solve quadratic equations with complex solutions.

Future Learning

During precalculus, students will take the learning of the complex numbers to divide complex numbers and will graph on the complex plane. Operations and graphing will continue into calculus and college trigonometry. Some occupations that will use the concept of complex numbers are farmers, ranchers, agricultural managers, insurance underwriters, and medical fields.

The world and its problems cannot be always be solved with real numbers or fractions. Many situations will require the use of complex numbers. Electronic computation technologies provide opportunities for students to work on realistic problems and perform difficult computations.

Additional Findings

According to *Principles and Standards for School Mathematics*, high school students should use real numbers and learn enough about complex numbers to interpret them as solutions to quadratic equations. They should fully understand the concept of a number system, how different numbers are related, and whether the properties in one system hold to another. (pp. 291–294)

The book also states that students should be able to solve problems involving complex numbers using a pencil in some cases and technology in all cases. (p. 395)

Introducing the Characteristics of Polynomial Functions Using Graphing

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Understand that polynomials form a closed system under addition, subtraction, and multiplication.
- Apply the Remainder Theorem for a polynomial.
- Use synthetic division to solve polynomials.
- Make a rough graph of polynomials using the zeros.
- Find key features of polynomials using graphs (by hand and using technology) and tables. [Functions: linear, quadratic square roots, cube roots, piecewise, absolute value, step, exponential, logarithm, trigonometry. Include minimum/maximum, symmetry, end behavior, periodicity, increasing, decreasing, positive, negative, period, midline and amplitude.]

Mathematical practices to be integrated

Model with mathematics.

- Use graphing calculator to estimate rate of change.
- Use tables and graphs to illustrate the features of polynomials.
- Find factors to solve of polynomials.

Use appropriate tools strategically.

- Use graphing calculator to identify zeros of polynomials.
- Use graph paper, straight edge, and writing utensils to make a rough draft to graph polynomials.

Look for and make use of structure.

- Use synthetic division to prove the Reminder Theorem.
- Prove polynomials are a closed set under the arithmetic operations of addition, subtraction, and multiplication.

Essential questions

- What are the similarities and differences between long division and synthetic division?
- How do graphs and tables help you find the key features of polynomials?
- How is division of polynomials connected to the Remainder Theorem?
- Why are zeros important in the graphing of polynomials
- What is the relationship between zeros and factors of polynomials?

Written Curriculum

Common Core State Standards for Mathematical Content

Arithmetic with Polynomials and Rational Expressions — A-APR

Perform arithmetic operations on polynomials [*Beyond quadratic*]

A-APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Understand the relationship between zeros and factors of polynomials

A-APR.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.

A-APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Interpreting Functions — F-IF

Interpret functions that arise in applications in terms of the context [*Emphasize selection of appropriate models*]

F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

F-IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

Analyze functions using different representations [*Focus on using key features to guide selection of appropriate type of model function*]

F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

- b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

7 Look for and make use of structure.

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Clarifying the Standards

Prior Learning

In grade 5, students graphed points on the coordinate plane to solve problems. In grade 6, students solved real-world problems by graphing points in four quadrants. In grade 7, proportional relationships were used to introduce the concept of slope. In grade 8, students worked with, found the characteristics, and mastered linear functions. In algebra I, students worked with, found the characteristics and mastered quadratic functions. In geometry, students work with slope to prove theorems about angles.

Current Learning

Polynomials are a critical area for algebra 2. Students study all facets of all polynomials functions using graphs and tables. The functions included are: square roots, cube roots, and piecewise-defined functions, step functions, absolute value functions, exponential functions, and logarithmic functions. The characteristics of all functions must include: intercepts, end behavior, minimums, maximums, symmetry, periodicity, increasing and decreasing, positive and negative, period, midline, and amplitude, where applicable.

Future Learning

Students will use concepts learned in this unit in the future to understand the characteristics of all functions. In precalculus, students will extend their graphing of rational functions, identify zeros and asymptotes when suitable factorizations are available, and show end behavior. Mastery of these concepts will be required in precalculus and advanced-placement calculus. They will also be used as students work in careers, such as: architects, engineers, funeral home directors, insurance underwriters, registered nurses, and scientists

Additional Findings

Students often struggle with "seeing" how a function can model a real-world problem. Using technology, teachers can help students model and understand mathematical concepts. "With utilities for symbol manipulation, graphing, and curve fitting and with programmable software and spreadsheets to represent iterative processes, students can model and analyze a wide range of phenomena. These mathematical tools can help students develop a deeper understanding of real-world phenomena. At the same time, working in real contexts may help students make sense of the underlying mathematical concepts and may foster an appreciation of those concepts . . . In helping high school students learn about the characteristics of particular classes of functions, teachers may find it helpful to compare and contrast situations that are modeled by functions from various classes." (*Principles and Standards for School Mathematics*, p. 297)

Algebra 2, Quarter 1, Unit 1.4

Transforming, Solving Using the Intersection of Linear Functions, and Solving for Zeros Using Graphs

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Explain process of reasoning for solving equations.
- Represent equations graphically (linear, polynomial, rational, absolute value, exponential, and logarithmic functions).
- Represent inequalities graphically (linear, polynomial, rational, absolute value, exponential, and logarithmic functions).
- Represent transformations of functions symbolically by showing key features of the graphs (linear, polynomial, rational, absolute value, exponential, and logarithmic functions).
- Graph (linear, polynomial, rational, absolute value, exponential, and logarithmic) functions.
- Identify the zeros of the functions (linear, polynomial, rational, absolute value, exponential, and logarithmic functions) (linear, polynomial, rational, absolute value, exponential, and logarithmic).
- Find the end behavior of the functions (linear, polynomial, rational, absolute value, exponential, and logarithmic functions).

Mathematical practices to be integrated

Model with mathematics.

- Use graphing calculator to find the point of intersection and understand the x value is the solution.
- Use graphing calculator to show transformations.
- Use graph paper and pencil to analyze mathematically to draw conclusions.

Use appropriate tools strategically.

- Use graph paper, pencil, and straightedge to solve equations graphically.
- Use graph paper, pencil, and straightedge to solve inequalities graphically.
- Use graphing calculator to solve equations graphically.
- Use graphing calculator to solve inequalities graphically.

Look for and make use of structure.

- Observe the similarities between the functions when using transformations.

Essential questions

- What are the similarities and differences of solving functions using systems of equations versus solving the function for the zero graphically?
- What are the similarities and differences in the transformations of functions?
- How are the solutions to equations and inequalities different?
- Why is the end behavior important to the study of functions?

Written Curriculum

Common Core State Standards for Mathematical Content

Reasoning with Equations and Inequalities — A-REI

Represent and solve equations and inequalities graphically [*Linear and exponential; learn as general principle*]

A-REI.11 Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★

Building Functions — F-BF

Build new functions from existing functions [*Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types*]

F-BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

Interpreting Functions — F-IF

Analyze functions using different representations [*Focus on using key features to guide selection of appropriate type of model function*]

F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

Common Core Standards for Mathematical Practice

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Clarifying the Standards

Prior Learning

In grade 1, students represented and solved problems using addition. In grade 2, students represented and solved problems using subtraction. In grade 3, students represented and solved problems using multiplication and division. In grade 4, students represented and interpreted data. In grade 5, students points on a coordinate plane. In grade 6, students reasoned about and solved one-variable equations and inequalities. In grade 7, students drew, constructed and described geometrical figures and described the relationships between them. In grade 8, students analyzed and solved linear equations and pairs of simultaneous linear equations. They defined, evaluated, and compared functions. In algebra 1, students understood solving equations as a process of reasoning and explained the reasoning. They represented and solved equations and inequalities graphically. In geometry, students experimented with similarity in transformations in the plane.

Current Learning

Transformation of functions is a critical area of algebra 2. Students generalize what they have learned about a variety of function families. They explore the effects of transformations on diverse functions and discover that their graphs always have the same effect. Students solve equations and inequalities graphically. Students graph transformations, identify the zeros, and find the end behavior of functions.

Future Learning

After learning to do transformations in algebra 2, students will build new functions from existing functions during precalculus. Students will analyze functions using different representations after learning about end behaviors in algebra 2. Fashion designers use translation of functions in their designs. Electronics and electrical installers are involved in translation of functions in areas like the oilfield, electricians, and mechanics. Computer technicians use the skills in maintenance and building of computers.

Additional Findings

Transformations of functions can cause some students to struggle if they do not realize that the behavior of all functions is the same. Students can use technology, such as graphing calculators, to represent and study the behavior of functions such as polynomial, exponential, rational, and periodic functions. "In high school, students should have opportunities to build on these earlier experiences, both deepening their understanding of relations and functions and expanding their repertoire of familiar functions." (*Principles and Standards for School Mathematics*, p. 297)